WAVES and SYMMETRY
Raman Centenary Symposium,
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Geometric Phase – Introductory Remarks

N. MUKUNDA
Centre for Theoretical Studies,
Indian Institute of Science, Bangalore 560 012, India.

It is a well known fact that complex numbers enter the mathematical formalism of quantum mechanics in a very fundamental way. Thus their use in this context is far from being a matter of mere convenience, as it is in, for instance, classical electromagnetic theory. For any quantum system, pure states can be superposed with one another using complex numbers as coefficients, and the results are pure states again. As an illustration, by considering all possible complex superpositions of two independent pure states, we obtain a whole S^2 – worth of pure states, rather than a set describable by, say, a "smaller" one-dimensional manifold. This appearance of complex numbers leads to many delicate phase-dependent quantum phenomena, including those of constructive and destructive interference of complex probability amplitudes.

In 1984 Michael Berry[1] made a very fundamental discovery relating to phases in quantum mechanics: at the end of a cyclic adiabatic Hamiltonian evolution governed by the time-dependent Schrodinger equation, while the physical state may return to its initial "value", the state vector need not, there being room for a difference in phase. Moreover part of this phase is basically geometrical rather than dynamical in character. In pure mathematics, certain results are frequently characterised as being "deep". To be deep is not necessarily the same as to be intricate or complicated, but to refer somehow to the very foundations of the subject. There is of course an unavoidable element of subjectivity in saying that a theorem is deep; it reminds one of Dirac's statement that "Mathematical beauty cannot be defined any more than beauty in art can be defined, but which people who study mathematics usually have no difficulty in appreciating". Berry's discovery is both deep and beautiful; today it is named in his honour as "The Berry Phase".

Soon after, many generalisations and applications of the Berry phase appeared: relaxing the adiabaticity condition, relating it to corresponding classical phenomena, tracing anomalies in quantum field theory to such origins and so on. What motivated a considerable amount of work in this country was the recognition and display by S. Ramaseshan and R. Nityananda[2] of the fact that something very closely analogous to the Berry phase had in fact been discovered by S. Pancharatnam[3] in the mid-fifties in the context of polarization optics. Later Berry himself re-evaluated and placed in perspective this early pioneering work done in India[4].

It should come as no surprise that many of the practical experimental demonstrations of these delicate phase effects on a macroscopic scale should have to do with propagation of light beams and cyclic changes of its state of polarization. Equally it should be understandable that many examples of the Berry phase and its generalizations can be best theoretically handled using the language of unitary group representations. The three papers put together in this part of this volume are, from all points of view, most appropriate: we have S. Ramaseshan tracing the historical connection between Pancharatnam's and Berry's discoveries; R. Simon reporting on the uses of the symplectic groups in the theory of the geometric phase in optics; and R. Bhandari describing a unified framework to handle optical experiments concerned with these questions. It is unfortunate that we have been unable to include here the text of Berry's own talk given at this session of the Symposium, due to factors beyond our control.

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