Exponential Diversity Achieving Spatio-Temporal Power Allocation Scheme for MIMO Fading Channels

K. Premkumar1, A. Rangarajan, Vinod Sharma  
Dept. of Electrical Communication Engineering  
Indian Institute of Science, Bangalore – 560 012, INDIA.  
e-mail: {prem, arun}@pal.ece.iisc.ernet.in, vinod@ece.iisc.ernet.in

Abstract — We analyze optimum (space-time) adaptive power transmission policies for Rayleigh fading MIMO channels when CSIT and CSIR are available. We show that our power allocation policy provides exponential diversity gain. BER ≤ \( \frac{1}{\alpha} \) for conventional space-time power allocation (STPA) for an uncoded system having \( n_t = 4 \) and \( n_r = 2 \). We observe that for the perfect CSIT case, our power allocation policy provides exponential order diversity gain for both the coded and uncoded systems which is substantially more than the conventional space-only (SOPA) [1] and uniform power allocation schemes. Also, when the quality of CSIT degrades (\( \rho \neq 1 \)), the exponential diversity is lost at high SNR for our policy. The interesting observation here is that we still achieve exponential diversity at low SNR (up to 9 dB in the systems we studied). In Fig. 2, we note that at \( \rho = 1 \) and \( P_c = 10^{-3} \) the gain in STPA compared to SOPA is 2.5 dB and at \( \rho = 0.99 \) the gain is 3.2 dB. The channel estimation in GSM is done by a 26-bit midamble sequence. A ML estimate of \( H \) from this sequence can provide \( \rho \) as large as 0.99 for a SNR of 10 dB. Furthermore, since the fading is i.i.d., interleaving will not improve the performance.

I. PERFECT/IMPERFECT CSIT
We consider a single user narrowband (flat fading) communication system employing \( n_t \) transmit antennas and \( n_r \) receive antennas. The channel between \( j^{th} \) receive antenna and \( j^{th} \) transmit antenna, \( h_{ij} \), is a complex Gaussian random variable (\( H = [h_{ij}] \) represents the channel). We assume i.i.d. Rayleigh fading from symbol to symbol and on each of the diversity branches. The additive noise, \( n_i \), is temporally and spatially white with mean zero, i.e., \( n_i \sim N_C(0, \sigma^2 I_{n_r}) \). We assume that \( H \) is the transmitter’s estimate of the channel.

We assume that \( H \) and \( H \) are jointly complex Gaussian with correlation \( \rho \). We assume perfect CSIR. \( H \) is used to get the optimal beamforming transmit weight vector \( w \) (the eigenvector of \( H H^H \) corresponding to its largest eigenvalue) and transmit power \( P(\gamma) \) for that symbol duration. The output of the matched filter sampled at symbol duration is given by \( y = \sqrt{P(\gamma)} H w + n \), where \( x \) is the transmitted symbol, \( \gamma = \|H w\|^2/\sigma^2 \) is the SNR, \( P(\gamma) \) is the transmit power, and \( \gamma = (\|H w\|^2/\sigma^2) \) is the estimate of \( \gamma \) at the transmitter.

The BER performance of the above system for the coherent BPSK signaling is given by \( P(\gamma) = Q(\sqrt{2\gamma P(\gamma)}) \). We minimize \( P(\gamma) \) subject to the average transmit power constraint. For the perfect CSIT case (\( \gamma = \gamma_0 \)), the optimization problem is

\[
\min_{\gamma \leq P(\gamma) \leq 0} \int_0^\infty \left( \sqrt{2\gamma P(\gamma)} - f_r(\gamma) d\gamma \right)
\]

where \( \gamma_0 \) is the solution of \( \int_0^\infty P(\gamma) f_r(\gamma) d\gamma = 1 \). For the imperfect CSIT case, replace \( \gamma \) by \( \gamma \) in Eqn. 1. It should be noted here that the transmitter should have the knowledge of the fading statistics to compute \( \gamma_0 \) and estimate of instantaneous fade values \( H \) to compute \( P(\gamma) \).

We evaluated the upper bounds of the BER for the perfect and imperfect CSIT cases and the results are tabulated below.

MISO: \( P_c \leq 0.5 e^{-\sqrt{\frac{\alpha}{\gamma}}} \)

Perfect CSIT

<table>
<thead>
<tr>
<th>BER vs avg. SNR for different values of ( \rho ) of MIMO (4,2) system.</th>
<th>BER vs avg. SNR of a Alamouti &amp; Conv. coded (G(D)) of (1+ ( D^2+1+D^2 )) (2,1) system</th>
</tr>
</thead>
</table>

REFERENCES