PHOTON-PHOTON TOTAL INELASTIC CROSS-SECTION

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Abstract

We discuss predictions for the total inelastic $\gamma\gamma$ cross-section and their model dependence on the input parameters. We compare results from a simple extension of the Regge Pomeron exchange model as well as predictions from the eikonalized mini-jet model with recent LEP data.

It is by now established that all total cross-sections, including photoproduction, rise as the c.m. energy of the colliding particles increases. So far a successful description of total cross-sections is obtained in the Regge/Pomeron exchange model [1], in which a Regge pole and a Pomeron are exchanged and total cross-sections are seen to first decrease and subsequently rise according to the expression

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\[
\sigma_{ab}^{\text{tot}} = Y_{ab}s^{-\eta} + X_{ab}s^\epsilon
\]

where \(\epsilon\) and \(\eta\) are related to the intercept at zero of the leading Regge trajectory and of the Pomeron, respectively \(\eta \approx 0.5\) and \(\epsilon \approx 0.08\). This parametrisation applies successfully \([1]\) to photoproduction, as shown in Fig. 1, and to the lower energy data on \(\gamma\gamma\) \([2]\). Assuming the hypothesis of factorization at the poles, one can make a prediction for \(\gamma\gamma\) total inelastic cross-section, using

\[
Y_{ab}^2 = Y_{aa}Y_{bb} \quad X_{ab}^2 = X_{aa}X_{bb}
\]

and extracting the coefficients \(X\) and \(Y\) from those for the fit to photoproduction and hadron-hadron data. In particular, using for \(\eta\) and \(\epsilon\) the average values from the Particle Data Group compilation \([3]\) and averaging among the \(pp\) and \(\bar{p}p\) coefficients, one can have a first check of the factorization hypothesis. Noticing that the coefficient \(Y\) from photoproduction data has a large error and that prediction from the Regge/Pomeron exchange model refer to total cross-sections rather than the inelastic ones, these predictions can be enlarged into a band as shown in Fig. 2.

An alternative model for the rise of all total cross-sections, relies on hard parton-parton scattering. It was suggested \([4]\) that hard collisions between elementary constituents of the colliding hadrons, the partons, could be responsible for this rise which starts around \(\sqrt{s} \geq 10 \div 20\) GeV. This suggestion has subsequently evolved into mini-jet models \([5]\), whose eikonal formulation satisfies unitarity while embodying the concepts of rising total cross-sections with rising jet cross-sections. For processes involving photons, the model has to incorporate \([6]\) the hadronisation probability \(P_{\gamma}^{\text{had}}\) for the photon to fluctuate itself into a hadronic state. The eikonalised mini–jet cross-section is then

\[
\sigma_{ab}^{\text{inel}} = P_{ab}^{\text{had}} \int d^2\vec{b}[1 - e^{n(b,s)}]
\]

with the average number of collisions at a given impact parameter \(\vec{b}\) given by

\[
n(b, s) = A_{ab}(b)(\sigma_{ab}^{\text{soft}} + \frac{1}{P_{ab}^{\text{had}}}\sigma_{ab}^{\text{jet}})
\]

In eqs. \((1, 2)\), \(P_{ab}^{\text{had}}\) is the probability that the colliding particles \(a, b\) are both in a hadronic state, \(A_{ab}(b)\) describes the transverse overlap of the partons in the two projectiles normalised to 1, \(\sigma_{ab}^{\text{soft}}\) is the non-perturbative part of
the cross-section from which the factor of $P_{ab}^{\text{had}}$ has already been factored out and $\sigma_{ab}^{\text{jet}}$ is the hard part of the cross-section. The basic statement of the mini-jet model for total cross-sections is that the rise in $\sigma_{ab}^{\text{jet}}$ drives the rise of $\sigma_{ab}^{\text{inel}}$ with energy. Letting

$$P_{\gamma p}^{\text{had}} = P_{\gamma}^{\text{had}} \quad \text{and} \quad P_{\gamma\gamma}^{\text{had}} \approx (P_{\gamma}^{\text{had}})^2$$

(3)

one can extrapolate the model from photoproduction to photon-photon collisions. The issue of total $\gamma\gamma$ cross-sections assumes an additional significance in view of the large potential backgrounds that Beamstrahlung photons could cause at future Linear Colliders [7]. Because the hadronic structure of the photon involves both a perturbative and nonperturbative part, it has been proposed [2, 8] to use a sum of eikonalized functions instead of eq.(1) in processes involving photons.

The predictions of the eikonalized mini-jet model for photon induced processes thus depend on 1) the assumption of one or more eikonal 2) the hard jet cross-section $\sigma_{jet} = \int_{p_{t\text{min}}}^{\infty} d^{2}p_{t}^{2} d^{2}p_{t}^{2}$ which in turn depends on the minimum $p_{t}$ above which one can expect perturbative QCD to hold viz. $p_{t\text{min}}$ and the parton densities in the colliding particles $a$ and $b$, 3) the soft cross-section $\sigma_{ab}^{\text{soft}}$ 4) the overlap function $A_{ab}(b)$, defined as

$$A(b) = \frac{1}{(2\pi)^2} \int d^{2}\vec{q}F_{1}(q)F_{2}(q)e^{i\vec{q} \cdot \vec{b}}$$

(4)

where $F$ is the Fourier transform of the $b$-distribution of partons in the colliding particles and 5) last, but not the least, $P_{ab}^{\text{had}}$.

In this note we shall restrict ourselves to a single eikonal. The hard jet cross-sections are calculated in LO perturbative QCD and use photonic parton densities GRV [9] calculated to the leading order. We determine $\sigma_{\gamma p}^{\text{soft}}$ from $\sigma_{\gamma p}^{\text{soft}}$ which in turn is determined by a fit to the photoproduction data. From inspection of the photoproduction data, one can assume that $\sigma_{\gamma p}^{\text{soft}}$ should contain both a constant and an energy decreasing term. Following the suggestion [8]

$$\sigma_{\gamma p}^{\text{soft}} = \sigma^{0} + \frac{A}{\sqrt{s}} + \frac{B}{s}$$

(5)

we then calculate values for $\sigma^{0}, A$ and $B$ from a best fit [10] to the low energy photoproduction data, starting with the Quark Parton Model ansatz.
For $\gamma \gamma$ collisions, we repeat the QPM suggestion and propose

$$\sigma_{\gamma \gamma}^{\text{soft}} = \frac{2}{3} \sigma_{\gamma p}^{\text{soft}}, \quad \text{i.e. } \sigma_{\gamma \gamma}^{0} = 20.8 \text{mb, } A_{\gamma \gamma} = 6.7 \text{ mb GeV}^{3/2}, B_{\gamma \gamma} = 25.3 \text{ mb GeV}^{3/2}$$

(6)

Whereas the effect of the uncertainties in the above three quantities on the predictions of the inelastic photoproduction and $\gamma \gamma$ cross-sections has been studied in literature to a fair extent \[2, 8, 11\] the effect of the other two has not been much discussed. In the original use of the eikonal model, the overlap function $A_{ab}(b)$ of eq.(4) is obtained using for $F$ the electromagnetic form factors. For protons this is given by the dipole expression

$$F_{\text{prot}}(q) = \left[ \frac{\nu^2}{q^2 + \nu^2} \right]^2$$

(7)

with $\nu^2 = 0.71 \text{ GeV}^2$. For photons a number of authors \[8, 12\], on the basis of Vector Meson Dominance, have assumed the same functional form as for pion, i.e. the pole expression

$$F_{\text{pion}}(q) = \frac{k_0^2}{q^2 + k_0^2} \quad \text{with} \quad k_0 = 0.735 \text{ GeV}.$$  

(8)

There also exists another possibility, i.e. that the $b$-space distribution of partons is the Fourier transform of their intrinsic transverse momentum distributions \[13\]. While for the proton this would correspond to use a Gaussian distribution instead of the dipole expression, eq.(7), for the photon one can argue that the intrinsic transverse momentum ansatz \[14\] would imply the use of a different value of the parameter $k_0$ \[15\] in the pole expression for the form factor. By varying $k_0$ one can then explore both the intrinsic transverse distribution case and the form factor cum VMD hypothesis. Notice that the region most important to this calculation is for large values of the parameter $b$, where the overlap function changes trend, and is larger for smaller $k_0$ values.

Let us now look at $P_{\gamma}^{\text{had}}$. This is clearly expected to be $O(\alpha_{\text{em}})$. Based on Vector Meson Dominance one expects,

$$P_{\gamma}^{\text{had}} = P_{VMD} = \sum_{V=\rho, \omega, \phi} \frac{4\pi\alpha}{f_V^2} = \frac{1}{250}$$

(9)
Although in principle, $P_{\text{had}}$ is not a constant, for simplicity, we adopt here a fixed value of $1/204$, which includes a non-VMD contribution of $\approx 20\%$. Notice that a fixed value of $P_{\text{had}}$ can be absorbed into a redefinition of the parameter $k_o$ through a simple change of variables.

Having thus established the range of variability of the quantities involved in the calculation of total inelastic photonic cross sections, we can proceed to compare the predictions of the eikonalized minijet model with data. We use GRV (LO) densities and show the mini-jet result in Fig.1, using the form factor model for $A(b)$, i.e. eq. (4) with $k_o = 0.735 \text{ GeV}$. In the figures, we have not added the direct contribution, which will slightly increase the cross-section in the 10 GeV region. We observe that it is possible to include the high energy points using GRV densities and $p_{t\text{min}} = 2 \text{ GeV}$, but the low energy region would be better described by a smaller $p_{t\text{min}}$. This is the region where the rise, according to some authors, notably within the framework of the Dual Parton Model, is attributed to the so-called soft Pomeron.

We now apply the same criteria and parameter set used in $\gamma p$ collisions to the case of photon-photon collisions, i.e. $P_{h/\gamma} = 1/204$, $p_{t\text{min}} = 2 \text{ GeV}$ and $A(b)$ from eq. (4). A comparison with $\gamma\gamma$ data shows that although

(a) Fig.1: Total inelastic photon-proton cross-section
(b) Fig.2: Total inelastic photon-photon cross-section.
the value $k_0 = 0.735$, corresponding to the pion-factor, is compatible with the low energy data up to 10 GeV \[17\] within the limits established by the large errors involved, at higher energies \[18\] the best fit is obtained using a slightly larger value, i.e. $k_0 = 1$ GeV, and this is the one used in Fig.2. For comparison, we have also added mini-jet model predictions with SAS1 photon densities \[19\] and predictions (Pomeron/SaS) based on a Pomeron/Regge type parametrization\[2\].

References


[18] W. van Rossum, these proceedings.