The Dispersion-Velocity of Galactic Dark Matter Particles

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Abstract

The self-consistent spatial distribution of particles of Galactic dark matter is derived including their own gravitational potential, as also of that of the visible matter of the Galaxy. In order to reproduce the observed rotation curve of the Galaxy the value of the dispersion velocity of the dark matter particles, $\langle v^2 \rangle_{DM}^{1/2}$, should be $\sim 600$ km s$^{-1}$ or larger.

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More than 20 years ago, it was suggested [1] that weakly interacting particles of non-zero rest mass which decouple from radiation and matter early after the Big Bang would form an invisible gravitating background of dark matter (DM) around galactic systems. Even though at that time the only available candidates for these particles were the neutrinos of the muon and electron flavors, the idea itself became the paradigm under which the newly discovered particles like the tau-neutrino and newly hypothesised particles within the context of possible physics beyond the Standard Model of particle physics could be incorporated. Also, during the latter half of the intervening twenty years, we have witnessed a tremendous growth in the experimental effort towards direct detection of these particles in the laboratory. The experiments are aimed at observing the effects of the impact of mainly the more massive candidate particles of DM with targets maintained at cryogenic temperatures which facilitate the observation of the tiny amount of energy deposited in the process against the background generated by internal and external radioactivity and by the cosmic rays. These developments are reviewed in detail by Trimble [2], Primack, Seckel and Sadolet [3], Caldwell [4], and Price [5].

The interpretation of these experiments to derive constraints on the properties of the unknown particles constituting a halo of dark matter in and around the Galaxy requires assumptions about the density and spectrum of velocities of the DM particles in the solar neighbourhood. These parameters have been obtained thus far by describing the DM halo as a single component isothermal sphere which is truncated at a particular radius [5]. The normalization for the density of DM particles comes from an analysis originally suggested by Oort [6] in which the observed spatial- and velocity distribution of stars near the solar system indicate a DM density of $\sim 0.3\, \text{GeV cm}^{-3}$ in the solar neighbourhood; Bahcall [7] gives a detailed account of this procedure. The 3-dimensional dispersion velocity of the DM particles, $\langle v^2 \rangle^{1/2}_{\text{DM}}$, has not been determined, however. It is customary to take recourse to the virial result pertaining to an isotropic isothermal sphere [8] and set $\langle v^2 \rangle^{1/2}_{\text{DM}} = \sqrt{3} \Theta_{\infty}$, where $\Theta_{\infty}$ is the asymptotic value of the circular rotation speed. Since $\Theta_{\infty}$ for the Galaxy is not known, the usual practice is to assume that the rotation curve of the Galaxy $\Theta(R)$, is flat from $R \sim 5\, \text{kpc}$ out to $R \gg R_0 \approx 8.5\, \text{kpc}$ (here and below $R$ denotes the galactocentric distance in the plane of the Galaxy, $R_0$ being the sun’s position), and set $\Theta_{\infty} \approx \Theta(R_0) \approx 220\, \text{km s}^{-1}$, the rotation speed near the solar system. This yields $\langle v^2 \rangle^{1/2}_{\text{DM}} \approx 270\, \text{km s}^{-1}$, which is the value usually assumed in most studies of issues related to Galactic DM. However, as noted in the recent review by Fich and Tremaine [12], “Much of the data indicates that the rotation curve continues to rise beyond $R_0$”. Thus the estimate $\langle v^2 \rangle^{1/2}_{\text{DM}} \sim 270\, \text{km s}^{-1}$ derived by assuming $\Theta_{\infty} = \Theta(R_0)$ is uncertain. Moreover, the assumption of a pure isothermal sphere for the description of the dark matter halo neglects the possible deviation from spherical symmetry induced by the disk-like distribution of the visible matter.

Keeping these points in mind, we focus attention on the observed rotation curve of the Galaxy, and develop a theoretical framework, the salient features of which are: (a) A model for the Galaxy comprising of visible matter and particles of DM with a self-consistent inclusion of their gravitational interactions, and (b) Departure from spherical symmetry due to the disk-like distribution of the visible matter which will be treated as axially symmetric. The quantity $\langle v^2 \rangle^{1/2}_{\text{DM}}$ appears as a free parameter in our framework and is determined by comparing the theoretical rotation curve with the observed data.

We adopt well-established models to describe the density distribution of the normal
visible matter and the resulting gravitational potential. In this Letter we present our results for a two-component model of the visible matter consisting of a spheroidal bulge with density \( \rho_s(r) \), and an axisymmetric disk with density \( \rho_d(R, z) \):

\[
\rho_s(r) = \frac{\rho_0}{\left(1 + \frac{r^2}{a^2}\right)^{3/2}}, \\
\rho_d(R, z) = \frac{\Sigma_0}{2h} e^{-(R-R_0)/R_d} e^{-|z|/h}
\]

where \( r = (R^2 + z^2)^{1/2} \), and \( \Sigma_0 \equiv \int_{-\infty}^{\infty} \rho_d(R_0, z)dz \) is the disk surface density at the solar position, \( z \) being the vertical distance from the plane of the disk. The values of the parameters are given by \( a = 0.103 \) kpc, \( R_d = 3.5 \) kpc, \( h = 0.3 \) kpc, and \( \rho_s(R_0) = 7 \times 10^{-4} \) M\( \odot \) pc\(^{-3} \). (Note that the rotation curve in the outer regions of the Galaxy is relatively insensitive to the spheroid parameters). There are conflicting reports on the value of \( \Sigma_0 \): Whereas Kuijken and Gilmore (KG) suggest \( \Sigma_0 \sim 40 \) M\( \odot \) pc\(^{-2} \) on the basis of data on \( \sim 512 \) K-dwarf stars, Bahcall et al. in their reanalysis of essentially the same data suggest a number for \( \Sigma_0 \) which is about twice as large. In our calculations we consider values of \( \Sigma_0 \) in the range \( 40–80 \) M\( \odot \) pc\(^{-2} \). The estimate of the local surface density of the Galactic disk due to the identified matter such as visible stars is \( \sim 48 \pm 8 \) M\( \odot \) pc\(^{-2} \). Thus Bahcall et al’s kinematical estimate of \( \Sigma_0 \) seems to indicate the presence of a substantial amount of unseen matter in the Galactic disk, whereas KG’s estimate is consistent with no disk dark matter. (Note that analyses of Refs. are all based on 1-dimensional solutions to the Boltzmann equation, which, in the given situation, are strictly valid for an infinite disk only). In any case, the dark matter associated with the disk is likely to be dissipational in contrast to that constituting the extended halo which would be collisionless and non-dissipative. We are concerned with this latter type of dark matter in this paper. We use the conventional nomenclature “visible” to describe effectively the total matter associated with the disk and write the total visible matter density, \( \rho_v \), as \( \rho_v = \rho_s + \rho_d \), the corresponding potential being \( \Phi_v = \Phi_s + \Phi_d \). The expressions for the potentials \( \Phi_s \) and \( \Phi_d \) corresponding to the chosen forms of \( \rho_s \) and \( \rho_d \) are given in Refs.

Now, for the DM component, the exercise is to calculate the distribution of the DM particles by self-consistently including the effects of the self-gravitation of the DM particles themselves and the potential due to the total visible component specified above. The procedure we follow is analogous to the one developed earlier with this difference that we now have to contend with the axial symmetry of the potentials. Since the DM particles obey the steady-state collisionless Boltzmann equation, the assumption of Maxwellian phase-space density allows us to write the spatial density, \( \rho_{\text{DM}}(R, z) \), of DM as

\[
\rho_{\text{DM}}(R, z) = \rho_{\text{DM}}(0, 0) \exp\left[-\frac{3}{\langle v^2 \rangle_{\text{DM}}} \left\{ \left( \Phi_{\text{DM}}(R, z) - \Phi_{\text{DM}}(0, 0) \right) + \left( \Phi_v(R, z) - \Phi_v(0, 0) \right) \right\} \right],
\]

where the DM potential, \( \Phi_{\text{DM}}(R, z) \), satisfies the Poisson equation,

\[
\nabla^2 \Phi_{\text{DM}}(R, z) = 4\pi G \rho_{\text{DM}}(R, z).
\]
The solution of the coupled equations (3) and (4) for $\Phi_{\text{DM}}$ is effected through the iterative scheme ($n = 1, 2, 3, \ldots$

\[ \nabla^2 \phi_n(R, z) = 4\pi G \rho_{n-1}(R, z), \quad (5) \]

where $\rho_{n-1}(R, z)$ is equal to the r.h.s. of Eq.(3) with $\Phi_{\text{DM}}$ replaced by $\phi_{n-1}(R, z)$, and

\[ \{\phi_n(R, z) - \phi_0(0, 0)\} = 0 \]

is the initial choice for the iteration process. The quantities $\rho_{\text{DM}}(0, 0)$ and $\langle v^2 \rangle_{\text{DM}}^{1/2}$ are taken as free parameters.

Details of the iterative scheme and the numerical procedure are described elsewhere. After a few iterations (typically, $n \leq 10$) the potentials $\phi_n$ converge towards the desired potential $\Phi_{\text{DM}}$. We checked our numerical code against test equations whose exact solutions are known. We also check our numerical results for the actual equations (3) and (4) against analytical results for small and large values of $R$ and $z$.

Once $\Phi_{\text{DM}}$ has been calculated, the rotation curve, $\Theta(R)$, is obtained through the relation

\[ \Theta^2(R) = \left( R \frac{\partial}{\partial R} \left[ \Phi_{\text{DM}}(R, z) + \Phi_{v}(R, z) \right] \right)_{z=0}. \quad (6) \]

Note that the contribution of the visible disk to $\Theta^2(R)$ is proportional to its surface density [see Eq.(4-159) of Ref. [9]], while that of a perfect isothermal sphere is proportional to the square of the velocity dispersion of its particles [see Eq.(4-127b) of Ref. [9]].

The theoretical rotation curves thus obtained for various values of the parameters $\rho_{\text{DM}}(0, 0)$ and $\langle v^2 \rangle_{\text{DM}}^{1/2}$ are to be compared with observations [10,11,12] to ascertain the domain of the parameter space which is acceptable. This comparison is shown in Fig.4 for $\Sigma_0 = 80 M_\odot$ pc$^{-2}$ and $\rho_{\text{DM}}(0, 0) = 1$ GeV cm$^{-3}$. The value of 80 M$_\odot$ pc$^{-2}$ for $\Sigma_0$, it being the upper limit on the allowed value of $\Sigma_0$ in our calculation, gives us a conservative estimate of (i.e., a lower limit on) $\langle v^2 \rangle_{\text{DM}}^{1/2}$. This is because, for a given value of $\Theta$ at a given value of $R$, a lower value of the disk surface mass density ($\Sigma_0$) requires a higher value of $\langle v^2 \rangle_{\text{DM}}^{1/2}$ (for a fixed value of $\rho_{\text{DM}}(0, 0)$). Our choice of $\rho_{\text{DM}}(0, 0) \approx 1$ GeV cm$^{-3}$ is dictated by the constraint [9,10] that $\rho_{\text{DM}}(R_0, 0) \sim 0.3$ GeV cm$^{-3}$ and the need to fit the rotation curve. A slightly lower value of $\rho_{\text{DM}}(0, 0)$ generally requires higher values of $\langle v^2 \rangle_{\text{DM}}^{1/2}$ in order to satisfy the above constraint and to fit the rotation curve. In this sense, our choice of $\rho_{\text{DM}}(0, 0) \approx 1$ GeV cm$^{-3}$ yields, again, a lower limit to $\langle v^2 \rangle_{\text{DM}}^{1/2}$. A higher value of $\rho_{\text{DM}}(0, 0)$, on the other hand, can be consistent with the constraint $\rho_{\text{DM}}(R_0, 0) \sim 0.3$ GeV cm$^{-3}$ for sufficiently low values of $\langle v^2 \rangle_{\text{DM}}^{1/2}$; however, in this case, the rotation curve falls steeply beyond the solar circle and thus provides a poor fit to the rotation curve.

In order to determine (a lower limit to) the best-fit value of $\langle v^2 \rangle_{\text{DM}}^{1/2}$ we have calculated $\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\Theta_i(R_i) - \Theta_{i,o}(R_i)}{\sigma_i} \right)^2$ as a function of $\langle v^2 \rangle_{\text{DM}}^{1/2}$ (for $\Sigma_0 = 80$ and 40 M$_\odot$ pc$^{-2}$ and $\rho_{\text{DM}}(0, 0) = 1$ GeV cm$^{-3}$), where $N$ is the number of observational data points, $\Theta_i(R_i)$ and $\Theta_{i,o}(R_i)$ are the theoretical and observational value of the rotation speed, respectively, for the $i$th data point for which $R = R_i$, and $\sigma_i$ is the 1σ uncertainty in the measured value of $\Theta_{i,o}(R_i)$. We calculate the above $\chi^2$ for the entire data set for $R$ in the range $\sim (2–20)$ kpc as well as for the restricted data set for $R$ in the range $\sim (10–20)$ kpc in which the observed rotation curve data show a conspicuous rising trend. For $\Sigma_0 = 80$ M$_\odot$ pc$^{-2}$, both data set give a minimum $\chi^2$ at $\langle v^2 \rangle_{\text{DM}}^{1/2} \sim 600$ km s$^{-1}$. For $\Sigma_0 = 40$ M$_\odot$ pc$^{-2}$, the minimum of
the $\chi^2$ lies at $\langle v^2 \rangle_{\frac{1}{2}}^{\text{DM}} \sim 750 \text{ km s}^{-1}$ for the restricted data set while the minimum is beyond $900 \text{ km s}^{-1}$ for the full data set. From the above analysis we conclude that the lower limit on $\langle v^2 \rangle_{\frac{1}{2}}^{\text{DM}}$ is $\sim 600 \text{ km s}^{-1}$.

Notice from Fig.4 that for $\langle v^2 \rangle_{\frac{1}{2}}^{\text{DM}} \sim 300 \text{ km s}^{-1}$ the potential of the visible component concentrates the distribution of DM towards the centre, causing the rotation curve to fall below the observational data at large galactocentric distances. As the kinetic energy of the DM particles increases with increased value of $\langle v^2 \rangle_{\frac{1}{2}}^{\text{DM}}$ the particles are affected progressively less by the potentials and spread out farther. This causes the rotation curves to be elevated.

We thus see that the rms velocity of particles of DM needed to generate the observed rotation curve is higher than that adopted in a variety of discussions of DM [18]. Indeed, we had an inkling that this might be so, based on our analytic estimates made earlier in this context [19]. The implications of this result are multifarious:

- 1. Since the typical velocity of individual DM particles is higher by at least a factor of $\sim 2$ on the average, the energies they would deposit in the detectors would be higher by at least a factor $\sim 4$. This would make these events stand out against the background.

- 2. The higher velocities imply higher fluxes and the event rates would be increased by at least a factor of $\sim 2$.

- 3. When the observed pulse height spectrum in the detectors are reanalysed taking the above two points into account the existing bounds on the masses and other properties of dark matter particles would become substantially more stringent.

- 4. The higher velocities would also mean lower rates of capture by the Sun by accretion; consequently the flux of high energy neutrinos arising from their annihilations in the central regions of the Sun [20] is expected to be correspondingly smaller.

- 5. The large velocities would also imply an extended halo (with an estimated mass of $\sim 1.5 \times 10^{12} \text{ M}_\odot$ up to $\sim 100 \text{ kpc}$) whose influence on the dynamical motions within our Galaxy and on the Local Group, as also the tidal effects on the dwarf-spheroidals would become important. For example, this high temperature halo will impart stability to the disk according to the criterion derived by Peebles and Ostriker [21].

These issues are under study and will be reported elsewhere.
REFERENCES

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[18] As per the suggestions received from the referee we have repeated the calculations with the King’s model [9] which ensures that the density of dark matter vanishes at a finite distance. The bound on \( \langle v^2 \rangle^{1/2}_{\text{DM}} \) quoted in this paper remain unaffected by this change. A manuscript incorporating the density contours, mass distribution etc is under preparation.
FIGURES

FIG. 1. The theoretically calculated rotation curve of the Galaxy for various values of $\langle v^2 \rangle_{DM}^{1/2}$ compared with the available observational data [10,11,12]. All curves are for $\rho_{DM}(0,0) = 1 \text{ GeV cm}^{-3}$ and $\Sigma_0 = 80 \text{ M}_\odot \text{ pc}^{-2}$ (see text). The data and error bars for $R$ in the range $\sim (2-17) \text{ kpc}$ are from Fig. 3 of Ref. [11], and those for $R > 17 \text{ kpc}$ are from Fig. 2 of Ref. [12]. The data for $R$ below $\sim 2 \text{ kpc}$ are from Ref. [10].